Universidad Pedagógica y Tecnológica de Colombia

Educación Virtual

THE TWO HIGGS DOUBLET MODEL TYPE III

Física

Herberth Jesús Cárdenas Ramírez - hjcr69@hotmail.com

The Two Higgs Doublet Model Type III

Goldstone Theorem

When an exact global simmetry is spontaneously broken, i. e, it is not a simmetry of the phisical vacuum, the theory contains one massless scalar particle for each broken generator of the original simmetry group.

The Goldstone theorem is for theories with spontaneously broken global simmetries but does not hold for gauge theories. When a spontaneous simmetry breaking takes place in a gauge theory the so-called Higgs Mechanism operates .

When the simmetry of vacuum is spontaneously broken the Goldstone boson (massless particles) arise in the spectrum.

However, if the Lagrangian posseses a local gauge simmetry an interrelation among gauge and Goldstone bosons endows the former with a physical mass, while the latter dissapear from the spectrum, this phenomenon is called the Higgs Mechanism.

In order to see how this works let us consider a toy model describing a couple of self interacting complex scalar fields $\left(\phi \text{ and } \phi^{*}\right)$ **whose Lagrangian is local gauge invariant.** [3]

$$\mathrm{L} = \frac{1}{2}\left|D^{\mu}\phi\right| - V\left(\phi^{*}\phi\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad \textbf{(1)}$$

with

$$V\left(\phi\right) = -\frac{1}{2}\mu^{2}\left|\phi\right|^{2} + \frac{1}{4}\lambda^{2}\left(\phi^{*}\phi\right)^{2} \qquad \textbf{(2)}$$

where ϕ**,**

$$\phi = \phi_{1} + i\phi_{2} \qquad \textbf{(3)}$$

is a complex field and

$$D_\mu = \partial_\mu + iqA_\mu \quad ; \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \textbf{(4)}$$

This Lagrangian has already the local gauge invariance described by the simultaneous transformations

$$\phi(x) \rightarrow e^{-iq\lambda(x)}\phi(x) \quad , \quad A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \lambda(x) \qquad \textbf{(5)}$$

The imposition of locality generates the interaction of the complex scalar fields with a four vector field. If $\mu^2 > 0$, the simmetry of the Lagrangian is spontaneously broken.

Choosing a particular minimun

$$\langle \phi_1 \rangle = \frac{\mu}{\lambda} \equiv v \quad ; \quad \langle \phi_2 \rangle = 0 \qquad \textbf{(6)}$$

we say that the field $\langle \phi_1 \rangle$ has acquired a Vacuum Expectation Value (VEV)$\langle \phi_1 \rangle$. It is convenient to introduce new fields

$$\eta \equiv \phi_1 - v \quad ; \quad \xi \equiv \phi_2 \qquad \textbf{(7)}$$

and expanding the Lagrangian in terms of the news fields we obtain

$$\mathrm{L} = \left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2\eta^2\right] + \frac{1}{2}\left[(\partial_\mu \xi)(\partial^\mu \xi)\right]$$

$$+\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A_\mu A^\mu\right] - 2iqv(\partial_\mu \xi)A^\mu$$

$$+\left\{\begin{array}{l} q\left[\eta(\partial_\mu \xi) - \xi(\partial_\mu \eta)\right]A^\mu + vq^2(\eta A_\mu A^\mu) + \frac{q^2}{2}(\xi^2 + \eta^2)A_\mu A^\mu \\ -\lambda\mu(\eta^3 + \eta\xi^2) - \frac{\lambda^2}{4}(\eta^4 + 2\eta^2\xi^2 + \xi^4) \end{array}\right\}$$

$$+\frac{\mu^2 v^2}{4} \qquad \textbf{(8)}$$

The particle spectrum consist of

1. **A field η with mass $\sqrt{2}\mu$**
2. **A vector boson A_{μ}, that has acquired a mass qv>0 by means of the VEV.**
3. **A massless field ξ called the Goldstone boson.**

In the Lagrangian (8) a term of the form $\left(\partial_{\mu}\xi\right)A^{\mu}$ does not have clear interpretation in the Feynman formalism. The unwanted would be Goldstone field is remove out, by mean of the local gauge invariance of the Lagrangian.

Writinng Eq. (5) in terms of ϕ_1 y ϕ_2

$$\phi \to \phi' = e^{i\theta(x)}\phi = \left[\phi_1 \cos\theta(x) - \phi_2 \sin\theta(x)\right] + i\left[\phi_1 \sin\theta(x) - \phi_2 \cos\theta(x)\right] \qquad \textbf{(9)}$$

where $\theta(x) \equiv -q\lambda(x)$**, and using**

$$\theta(x) = \arctan\left[\frac{\phi_2(x)}{\phi_1(x)}\right] \qquad \textbf{(10)}$$

we get ϕ' **to be real.**

The gauge field transforms as $A'_\mu(x) = A_\mu(x) + \partial_\mu \lambda(x)$**. However, this gauge transformation does not affect the physical content of** $A_\mu(x)$ **so we drop the prime notation out from it.**

Using the local transformations definided by (5) and (9), (i.e. this particular gauge), the Lagrangian reads

$$\mathsf{L} = \left[\frac{1}{2}\left(\partial_\mu \eta\right)\left(\partial_\mu \eta\right) - \mu^2\eta^2\right] + \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A_\mu A^\mu\right]$$
$$+\left\{q^2 v\left(\eta A_\mu A^\mu\right) + \frac{q^2}{2}\eta^2 A_\mu A^\mu - \lambda\mu\eta^3 - \frac{\lambda^2}{4}\eta^4\right\} + \mu^2 v^2$$

(11)

so we got rid of the massless field ξ **and its interactions, specially the "disgusting" term** $\left(\partial_\mu \xi\right)A^\mu$**. On the other hand, we are left with a massive scalar field** η **(a Higgs particle) and a massive four vector field** A_μ **(a massive "foton").**

Generically said that the photon has "eaten" the would be Goldstone boson ξ **in order to acquire mass. This result is know as Higgss mecanism.**

The SM of particles physics, picks up the ideas of local gauge invariance and SSB to implement a Higgs mechanism. Specifically, the simmetry breaking is implemented by introducing a scalar doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad ; \quad \Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \qquad \textbf{(12)}$$

It transforms as an $SU(2)_L$ **doublet, thus the hypercharge is Y= 1. In order to induce the SSB the doublet should acquire a VEV different from zero**

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \qquad \textbf{(13)}$$

The Higgs Doublet Model (2HDM) consist of adding a second Higgs doublet with the same quantum numbers as the first one

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \hline \phi_1^o \end{pmatrix} \quad ; \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \hline \phi_2^o \end{pmatrix} \qquad \textbf{(14)}$$

with hypercharges $Y_1 = Y_2 = 1$**, in general both doublets could acquire VEV**

$$\langle \Phi_1 \rangle = \frac{v_1}{\sqrt{2}} \quad ; \langle \Phi_2 \rangle = \frac{v_2}{\sqrt{2}} e^{i\phi} \qquad \textbf{(15)}$$

The most general gauge invariant Lagrangian that couples the Higgs field to fermions reads

$$-L_Y = \eta_{ij}^{U,0} \bar{Q}_{iL}^0 \tilde{\Phi}_1 U_{jR}^0 + \eta_{ij}^{D,0} \bar{Q}_{iL}^0 \Phi_1 D_{jR}^0 + \xi_{ij}^{U,0} \bar{Q}_{iL}^0 \tilde{\Phi}_2 U_{jR}^0$$

$$+ \xi_{ij}^{D,0} \bar{Q}_{iL}^0 \Phi_2 D_{jR}^0 + \eta_{ij}^{E,0} \tilde{l}_{iL}^0 \Phi_1 E_{jR}^0 + \xi_{ij}^{E,0} \tilde{l}_{iL}^0 \Phi_2 E_{jR}^0 + h.c., \qquad \textbf{(16)}$$

Where :

$\phi_{1,2}$ **represent the Higgs doublets,** $\tilde{\Phi}_{1,2} \equiv i\sigma_{1,2}\Phi_{1,2}$**,**

η^0_{ij} **and** ξ^0_{ij} **are non diagonal 3×3 matrices and i,j denote family indices.**

D^0_R **refers to the three down type weak isospin quark singlets** $D^0_R \equiv \left(d^0_R, s^0_R, b^0_R\right)$**,**

U **refers to the three down type weak isospin quark singlets** $U^0_R \equiv \left(u^0_R, c^0_R, t^0_R\right)$ **and** E^0_R **to the three charged leptons.**

Finally $\bar{Q}_{iL}, \bar{l}_{iL},$ **denote the quark and lepton weak isospin left handed doublets respectively. The superscript "0" indicates that the fields are not mass eigenestates yet.**

The model type III consists of taking into account all terms in Lagrangian (15). In the case of the model type III, we are able to make rotation of the doublets in such way that only one of the doublets acquire VEV. Therefore we can asume without any loss of generality that

$$\langle\Phi_1\rangle = v/\sqrt{2}, \quad \langle\Phi_2\rangle = 0 \tag{17}$$

References

[1] S. F. Novaes, Standard model: An Introduction, arXiv:hep-ph/0001283.

[2] M. Herrero, The Standard Model, arXivhep-ph/9812242.

[3] R. A. Díaz, Phenomenological Análisis of the Higgs Doublet Model, arXiv:hep-ph/o212237.