The Diamond Lemma

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May 18th, 2011

Abstract

The Diamond Lemma provides a general method in order to prove that certain sets are bases of algebras which are defined in terms of generators and relations. For instance, the Poincaré-Birkhoff-Witt theorem for enveloping algebras can be derived from it. We present some examples that illustrate the usefulness of this lemma. These examples are of interest for modern mathematical physics. The classes of rings we consider include as a special cases: quantum plane, algebra of q-differential operators, (quantum) Heisenberg and Weyl algebras, (quantum) enveloping algebra of the Lie algebra and others.

Key words: Poincaré-Birkhoff-Witt theorem, monomials, generators.

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